Problems of existence, regularity and relaxation under slow growth conditions: the one dimensional case.

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The classical results on problems of existence of minima, their regularity and also on the relaxation of integral functionals are based on the coercivity assumption on the lagrangean. In [1] a slow growth assumption was introduced for the one-dimensional case: it implies the superlinear case and includes some classes of functions with linear growth. In that paper the existence of Lipschitz minimizers was proved for a particular class of lagrangeans: we assumed that the langrangean does not depend on time, it is of sum type, smooth and convex with respect to the velocity. More recently, in [2] and [3] the authors extended this results to autonomous integrands, including the non-smooth and non-convex case. In particular the non-convex case is approched via a relaxation result. In [4] we considered the non continuous case: we just assume the measurability of the lagrangean. This case is not simply a generalization of the previous one: the relaxation result is not true anymore (in [5] can be found an example) and different kinds of growths have to be considered. The non-autonomous case is studied in [6]. An example shows that in order to get Lipschtz regularity of minima we need some conditions from above on the growth of the lagrangean with respect the time variable.

[1] Cellina, Arrigo; Treu, Giulia; Zagatti, Sandro *On the minimum problem for a class of non-coercive functionals.* J. Differ. Equations 127, No.1, 225-262 (1996).

[2] Cellina, Arrigo *The classical problem of the calculus of variations in the autonomous case: relaxation and Lipschitzianity of solutions.*
Trans. Am. Math. Soc. 356, No.1, 415-426 (2004).

[3] Cellina, Arrigo; Ferriero, Alessandro. Existence of Lipschitzian solutions to the classical problem of the calculus of variations in the autonomous case. *Ann. Inst. H. Poincaré Anal. Non Linéaire* **20** (2003), no. 6, 911--919

[4] Mariconda, Carlo; Treu, Giulia *Lipschitz regularity of minima under slow growth conditions: the non continuous and non- convex case.* to appear in Calc. Var.

[5] Mariconda, Carlo; Treu, Giulia *A relaxation result for autonomous integral functionals with discontinuous non-coercive integrand.*
ESAIM, Control Optim. Calc. Var. 10, 201-210 (2004)

[6] Treu, Giulia *Existence and Lipschitz regularity for one dimensional problems of the Calculus of Variations with slow growth: the non autonomous case* **in preparation**