A new direct optimization method for problems of the calculus of variations and optimal control

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Optimal control problems are usually addressed with the help of the famous Pontryagin Maximum Principle (PMP) which gives a generalization of the classic Euler-Lagrange and Weierstrass necessary optimality conditions of the calculus of variations. Success in applying the PMP permits to obtain candidates for a local minimum. In 1967 a direct method, which permits to obtain global minimizers directly, without using the necessary conditions, was given by Leitmann [4]. Since then, several extensions of Leitmann's method were obtained (see [2] and references therein). Leitmann's approach is connected, as showed by Carlson [1], with "Caratheodory's royal road of the Calculus of variations". In our work we propose a related but different direct approach to problems of the calculus of variations and optimal control, which permit to obtain global minima directly, without recourse to needle variations and necessary conditions. Our method is inspired on the classical Noether's theorem and its recent extensions to optimal control [5]. We make use of the symmetries of the problem, considering parameter-invariance transformations and substituting the original problem by a parameter-family of optimal control problems. Parameters are then fixed in order to make the problem trivial in some sense. Finally, by applying the inverse of the chosen invariance-transformation, we get the global minimizer for the original problem. A crucial step in the application of the proposed method resides in discovering the group of diffeomorfisms that define the invariance variational symmetries of the problem. For that, the Maple package introduced in [3] is useful. Our method applies to a sufficiently rich class of problems, which include interesting problems of sub-Riemannian geometry. We illustrate the method by solving problems not covered by the previous results.

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