Homogenization of the Sobolev trace constant

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We present homogenization problems for the best constant in the Sobolev trace embedding $W^{1,p}(\Omega) \hookrightarrow L^q(\partial\Omega)$ in a bounded smooth domain when the boundary is perturbed by adding an oscillation. We find that there exists a critical size of the amplitude of the oscillations for which the limit problem has a weight on the boundary. For sizes larger than critical the best trace constant goes to zero and for sizes smaller than critical it converges to the best constant in the domain without perturbations.

Moreover, we study the best constant of the Sobolev trace embedding in rapidly oscillating periodic media. Here we consider spaces with weights that are periodic in space. We find that extremals for this embedding converge to a solution of an homogenized limit problem and the best trace constant converges to a homogenized best trace constant. Also, we homogenize by considering a domain with holes inside the domain periodically distributed where we impose that the involved functions vanish. We find that there exists a critical size of the holes for which the limit problem has an extra term. For sizes larger than critical the best trace constant diverges to infinity and for sizes smaller than critical it converges to the best constant in the domain without holes. Finally, we prove some homogenization results when the holes are located on the boundary of the domain. In this case the critical size is different from the critical size for holes inside the domain and moreover for the critical size an extra term appears on the boundary.