Quasiconvexification for a 1-d hyperbolic optimal design problem.

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Optimal design problems have been extensively studied under a stationary perspective, for linear elliptic state equations. In this contribution we analyze a dynamic case, we seek the time-dependent optimal layout of two isotropic materials on a 1-d domain Ω by minimizing a functional depending quadratically on the gradient of the state.

$$\min_{\chi} \int_0^T \int_\Omega \left[u_t^2(t,x) + u_x^2(t,x)\right] \, dx \, dt$$

where u is the unique solution of

$$\begin{aligned} u_{tt} - ([\alpha\chi + \beta(1-\chi)]u_x)_x = 0 \quad & \text{in} \quad (0,T)\times\Omega, \\ u(0,x) = u_0(x), \ u_t(0,x) = u_1(x) \quad & \text{in} \quad \Omega, \\ u(t,x) = f(t), \quad & \text{in} \quad [0,T]\times\partial\Omega, \end{aligned}$$

As it is typical in this kind of problems, it is ill-posed in the sense that there is not an optimal design. We therefore examine relaxation by using the representation of two-dimensional ($(x,t) \in \mathbb{R}^2$) divergence free vector fields as rotated gradients. By means of gradient Young measures, we transform the original optimal design problem into a non-convex vector variational problem, for which we can compute an explicit form of the "*constrained quasiconvexification* " of the cost density. Moreover, this quasiconvexification is recovered by first or second-order laminates which give us the optimal microstructure at every point. After this relaxation we analyze the quasiconvexificated problem and propose some numerical simulations.