# Existence of scalar minimizers for nonconvex 1-dim integrals of the calculus of variations

Clara Carlota and António Ornelas

CIMA-UE, Évora, Portugal

**Abstract**

We prove existence of minimizers for the nonconvex integral

$$\int_a^b l\left(x\left(t\right), x'\left(t\right)\right) dt$$

among the $AC$ functions $x:[a,b]\to\mathbb{R}$ with $x\left(a\right)=A$, $x\left(b\right)=B$. The lagrangian $l:\mathbb{R}\times\mathbb{R}\to[0,+\infty]$ is assumed $\mathcal{L}\otimes\mathcal{B}$-measurable with coercive growth, having $l\left(s,\cdot\right)$ lsc, and $l^{**}\left(\cdot\right)$ lsc at $\left(s,0\right)$, $\forall s$. (Here $l^{**}\left(s,\cdot\right)$ is the closed convex hull of $l\left(s,\cdot\right)$.)

Besides these basic hypotheses we impose an extra hypothesis, to be chosen among several possibilities (e.g. zero-convexity).