

TABLAS

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*Tabla de relaciones trigonométricas.*

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$	$\cos(\alpha - \beta) + \cos(\alpha + \beta) = 2 \cos \alpha \cos \beta$
$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	$\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2 \sin \alpha \sin \beta$
$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$	$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta$
$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$	$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$
$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$	$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
$\sin 2\alpha = 2 \cos \alpha \sin \alpha$	$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$
$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$	$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$
$\operatorname{tg} 2\alpha = 2 \frac{\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$	$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$	$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$	Cuando $s \in [-\pi, \pi] \Rightarrow \left  \sin \frac{s}{2} \right  \geq \left  \frac{2}{\pi} \frac{s}{2} \right  \geq \left  \frac{s}{\pi} \right $
$\operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$	Si $\operatorname{tg} \frac{\alpha}{2} = t$ , $\cos \alpha = \frac{1 - t^2}{1 + t^2}$ ; $\sin \alpha = \frac{2t}{1 + t^2}$ ; $\tan(\alpha) = \frac{2t}{1 - t^2}$
	$d\alpha = \frac{2dt}{1 + t^2}$

*Tabla de trigonometría hiperbólica*

$\operatorname{senh} x = \frac{e^x - e^{-x}}{2}$	$\frac{d}{dx} \operatorname{senh} x = \cosh x$	$\cosh^2 x - \operatorname{senh}^2 x = 1$
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\frac{d}{dx} \cosh x = \operatorname{senh} x$	
$\operatorname{tanh} x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	$\frac{d}{dx} \operatorname{tanh} x = 1 - \operatorname{tanh}^2 x$	

### TABLA DE DERIVADAS

En adelante se supondrá  $y = f(x)$  y  $u = g(x)$ .

Función	Derivada	Función	Derivada
$y = K$ ( $K$ constante)	$y' = 0$	$y = \operatorname{tg} x$	$y' = 1 + \tan^2 x = \frac{1}{\cos^2 x}$
$y = x$	$y' = 1$	$y = \operatorname{tg} u$	$y' = u' (1 + \tan^2 u)$
$y = x^n$	$y' = nx^{n-1}$	$y = \cot x$	$y' = - (1 + \cot^2 x) = \frac{-1}{\sin^2 x}$
$y = u^n$	$y' = nu' u^{n-1}$	$y = \cot u$	$y' = -u' (1 + \cot^2 u)$
$y = \sqrt{x}$	$y' = \frac{1}{2\sqrt{x}}$	$y = \sec x$	$y' = \sec x \tan x$
$y = \sqrt{u}$	$y' = \frac{u'}{2\sqrt{u}}$	$y = \sec u$	$y' = u' \sec u \tan u$
$y = \sqrt[n]{x}$	$y' = \frac{1}{n\sqrt[n]{x^{n-1}}}$	$y = \operatorname{cosec} x$	$y' = -\operatorname{cosec} x \cot x$
$y = \sqrt[n]{u}$	$y' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$	$y = \operatorname{cosec} u$	$y' = -u' \operatorname{cosec} u \cot u$
$y = e^x$	$y' = e^x$	$y = \operatorname{arcsen} x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = e^u$	$y' = u' e^u$	$y = \operatorname{arcsen} u$	$y' = \frac{u'}{\sqrt{1-u^2}}$
$y = a^x$	$y' = a^x \ln a$	$y = \arccos x$	$y' = \frac{-1}{\sqrt{1-x^2}}$
$y = a^u$	$y' = u' a^u \ln a$	$y = \arccos u$	$y' = \frac{-u'}{\sqrt{1-u^2}}$
$y = \ln x$	$y' = \frac{1}{x}$	$y = \operatorname{arctg} x$	$y' = \frac{1}{1+x^2}$
$y = \ln u$	$y' = \frac{u'}{u}$	$y = \operatorname{arctg} u$	$y' = \frac{1}{1+u^2}$
$y = \log_a x$	$y' = \frac{1}{x} \log_a e$	$y = \operatorname{arccot} x$	$y' = \frac{-1}{1+x^2}$
$y = \log_a u$	$y' = \frac{u'}{u} \log_a e$	$y = \operatorname{arccot} u$	$y' = \frac{-u'}{1+u^2}$
$y = \operatorname{sen} x$	$y' = \cos x$	$y = \operatorname{arcsec} x$	$y' = \frac{1}{x\sqrt{x^2-1}}$
$y = \operatorname{sen} u$	$y' = u' \cos u$	$y = \operatorname{arcsec} u$	$y' = \frac{u'}{u\sqrt{u^2-1}}$
$y = \cos x$	$y' = -\operatorname{sen} x$	$y = \operatorname{arccosec} x$	$y' = \frac{-1}{x\sqrt{x^2-1}}$
$y = \cos u$	$y' = -u' \operatorname{sen} u$	$y = \operatorname{arccosec} u$	$y' = \frac{-u'}{u\sqrt{u^2-1}}$

## TABLA DE INTEGRALES

De aquí en adelante se supondrá  $u = g(x)$ ,  $c$  constante.

$\int dx = x + c$	$\int u^n u' dx = \frac{u^{n+1}}{(n+1)} + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$\int \frac{u}{u'} dx = \ln u  + c$
$\int \frac{1}{x} dx = \ln x  + c$	$\int u' e^u dx = e^u + c$
$\int e^x dx = e^x + c$	$\int a^u u' dx = \frac{1}{\ln a} a^u + c$
$\int a^x dx = \frac{1}{\ln a} a^x + c$	$\int \cos u u' dx = \sen u + c$
$\int \cos x dx = \sen x + c$	$\int \sen u u' dx = -\cos u + c$
$\int \sen x dx = -\cos x + c$	$\int \frac{u'}{\cos^2 u} dx = \tg u + c$
$\int \frac{1}{\cos^2 x} dx = \tg x + c$	$\int \frac{u'}{\sen^2 u} dx = -\cot u + c$
$\int \frac{1}{\sen^2 x} dx = -\cot x + c$	$\int \frac{u'}{\sqrt{1-u^2}} dx = \arcsen u + c$
$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsen x + c$	$\int \frac{-u'}{\sqrt{1-u^2}} dx = \arccos u + c$
$\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos x + c$	$\int \frac{u'}{1+u^2} dx = \arctg u + c$
$\int \frac{1}{1+x^2} dx = \arctg x + c$	$\int \frac{-u'}{1+u^2} dx = \arccot u + c$
$\int \frac{-1}{1+x^2} dx = \arccot x + c$	

## CÓNICAS

### 1.- Circunferencia

Ecuación para la circunferencia con centro  $(a, b)$  y radio  $r$

$$(x-a)^2 + (y-b)^2 = r^2$$

### 2.- Elipse.

Ecuación de la elipse con centro en  $(0, 0)$  y semiejes con medidas  $a$  y  $b$ .

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

### 3.- Hipérbola

a) Ecuaciones respecto a los ejes

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

b) Ecuaciones respecto a las asíntotas

$$x'y' = k.$$

### 4.- Parábola

$$y = ax^2 + bx + c.$$

## 1 Estimadores de los parámetros $a$ y $b$

Distribuciones de los estimadores insesgados de los parámetros  $a$  y  $b$ .

$$\hat{a} \equiv N \left( a, \frac{\sigma}{\sqrt{n}} \sqrt{1 + \frac{\bar{x}^2}{\sigma_x^2}} \right) \quad (1)$$

$$\hat{b} \equiv N \left( b, \frac{\sigma}{\sqrt{n\sigma_x^2}} \right) \quad (2)$$

Cuando  $\sigma^2$  es desconocida, al usar el Teorema Central de Límite en vez de  $\sigma^2$  se considera  $S_R^2$  con  $S_R^2 = \frac{1}{n-2} \sum_{i=1}^n (y_i - \hat{y}_i)^2$ .

## 2 Intervalos de confianza para los parámetros $a$ y $b$

Intervalo de confianza a  $1 - \alpha$  de confianza para el parámetro  $a$  con  $\sigma^2$  desconocido:

$$\left[ \hat{a} \pm t_{n-2,\alpha/2} \frac{S_R}{\sqrt{n}} \sqrt{1 + \frac{\bar{x}^2}{\sigma_x^2}} \right]$$

Intervalo de confianza a  $1 - \alpha$  de confianza para el parámetro  $b$  con  $\sigma^2$  desconocido:

$$\left[ \hat{b} \pm t_{n-2,\alpha/2} \frac{S_R}{\sqrt{n\sigma_x^2}} \right]$$

## 3 Contrastes de Hipótesis para el parámetro $b$

$$H_0 : b = 0 \text{ frente } H_1 : b \neq 0 \rightarrow \mathcal{R} = \{|t| > t_{n-2,\alpha/2}\}$$

$$H_0 : b \geq 0 \text{ frente } H_1 : b < 0 \rightarrow \mathcal{R} = \{t < t_{n-2,1-\alpha}\} \quad (3)$$

$$H_0 : b \leq 0 \text{ frente } H_1 : b > 0 \rightarrow \mathcal{R} = \{t > t_{n-2,\alpha}\}$$

donde  $t = \frac{\hat{b}}{S_R / \sqrt{n\sigma_x^2}}$