Growth models, the entropy method, examples from biology

Benoit Perthame

Laboratoire J.-L. Lions, Univ. P. et M. Curie,
Institut Universitaire de France,

benoit.perthame@upmc.fr

Abstract

Biological applications lead naturally to birth and death processes that can be described by zeroth order terms. They also lead to models where several structuring variables enter (age, size of individuals, physiological character or level of some proteins for cells) and several balance laws combine together (number of individuals, total biomass, total protein content) but no significant conservation law follows. Then the dynamics is strongly driven by these birth and death processes, more than by flux exchanges.

In this talk, we introduce the notion of General Relative Entropy Inequality that applies to PDEs that are not in conservation form. We show how the eigenelements come in the definition of the entropy and we give several types of applications of the General Relative Entropy Inequality: (i) a priori estimates and existence of solutions in a natural space, (ii) long time asymptotic to a steady state (is Poincar é inequality so useful for exponential rate of convergence to a steady state?), (iii) attraction to periodic solutions. This last point is motivated by the question: why is tumor growth favorized by a loss of circadian control?)

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